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A review of T-stress calculation methods in fracture mechanics computation

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Abstract

An overview of past research on T-stress is presented in this paper, we provides some critical review of the history and state of two elastic fracture mechanics and relationship to crack paths stability. The importance of the global approach with two parameters (K-T) or (K-A₃) in the analysis of the linear elastic fracture mechanics, based on the effect of confinement according to the work of Williams, is presented in the field of CT specimen in mode I for the API X52 steel. The objective is to propose a numerical study, defining the T-stress, by applying the finite element method, in 2D, using the software ANSYS 15.0. Thereafter, we propose the method inspired from the volumetric approach developed by Pluvinage, this method is based on the determination of the effective stress T_{ef} over an effective distance X_{ef} ahead of the crack tip. Finally, it concludes with a discussion critical of methods of calculated the T-stress.

Key words: *Constraint, T-stress, Effective Distance, Crack, Finite Element Analysis.*

1. Introduction

In fracture mechanics most interest problems are focused on the determination of the stress intensity factors K introduced by Irwin in 1948 [1,2]. The K describes the singular stress field ahead of a crack tip and governs the fracture of structures when a critical stress intensity factor is reached. In practice, there is always a region around the crack tip where plastic deformation, finite strain and damage occur. Consequently, the stresses do not follow just the

singular stress term inside this region and generally are leveled off due to damage of the material.

This approach requires that constraint in the test specimen approximate that of the structure to provide an “effective” toughness for use in a structural integrity assessment. The appropriate constraint is achieved by matching thickness and crack depth between specimen and structure. Experimental studies [2,3] demonstrate the validity of this approach. These studies show that the use of geometry

dependent fracture toughness values allows more accurate prediction of the fracture performance of structures then it is possible to conventional fracture mechanics.

The importance of the two-parameter approach in linear elastic fracture mechanics analysis is increasingly being recognized for fracture assessments in engineering applications. The consideration of the second parameter, namely, the elastic T-stress, allows estimating the level of constraint at a crack or notch tip.

The most important development is the nonvanishing parameters of Williams [3] equations. The second term is called the T-stress. The value of T_{xx} , or simply T , is constant stresses acting parallel to the crack line in the direction xx with a magnitude proportional to the gross stress in the vicinity of the crack. The third term A_3 is sometimes used as a transferability parameter like the T-stress.

Analytical and experimental studies have showed that the T parameter can be used as a measure of constraint for contained yielding; see for example Sumpter [4]. Chao et al. [5] and Hancock et al. [6] have shown that fracture toughness increases when $(-T)$ increases. In literature, many authors focused on the second parameter are the so-called T-Stress term which describes a constant stress parallel to the crack direction [7-17]. The non-singular term T represents a tension (or compression) stress. Positive T-stress strengthens the level of crack tip stress triaxiality and leads to high crack-tip constraint while negative T-stress leads to the loss of constraint. The value of T is sensitive to loading mode [18-21], specimen geometry [22-25], specimen and crack sizes [21], the T-stress increases from high negative value to low negative or positive values when specimen loading mode and geometry change from tension to bending. Sherry et al [26] indicates that the stress intensity factor over T ratio increases non linearly with non dimensional crack length. Rice [27], Larsson and Carlsson [28] have shown that sign and

magnitude of the T-stress substantially change the size and shape of the plane strain crack tip plastic zone. Positive or negative the T-stress increases the plastic zone size comparing with no T-stress situation. In plane strain, plastic zone is oriented along crack extension for $T > 0$ and in opposite sense when $T < 0$. It has been noted that in the Paris law regime, fatigue crack growth rate decreases when T increase [25]. Analytical and experimental studies show that the T-stress can be used as a measure of constraint ahead of the crack tip. Sumpter [4], Chao et al [29] and Hancock et al [6] have shown that the fracture toughness increases when $(-T)$ increases. It has been seen that the T-stress has an influence on crack propagation after initiation [13]. Negative T-stress values stabilise crack path. In opposite, positive T-stress value induces crack bifurcation [25]. A number of methods for obtaining T for a variety of loading conditions and geometries have been developed over the last 42 years. Some of the major methods are briefly described for the determination of T-stress solutions the following methods were applied; westergard stress function [30] the williams (Airy) stress function [3] the Green's function method [31] and the principle of superposition used by [32-37]. Other methods given by Williams [3] (1957)), Obtained the displacement and stress fields at the vicinity of a two-dimensional crack tip by the Eigen-function expansion method. Several numerical works used William's equation for obtaining the T-stress; the Stress Difference Method (SDM) of Yang et al. [23], Chao method [24] and the Displacement Method, Ayatollahi et al. [22].

Several authors disputed about how and at what distance are taking the values of the T-stress. Yang et al. [23] proposed the Stress Difference Method (SDM) to compute T -stress at crack tip. Chao et al. [24], from the numerical data, indicate that near the crack-tip, there exists nearly constant T-stress value. Kardomateas et al. [38] and Sherry et al. [26] they told that in practice it is seen that FE results are not

acceptable unless a large number of elements are used to simulate the crack tip zone. Ayatollahi et al. [22], indicate that the improved method of obtaining the T stress a reasonable distance from the crack tip. Without recourse to much mesh refinement is to use the displacements along the crack faces. Maleski et al. [21] has determined a modified stress difference method to calculate the T-stress, with extrapolation method. The above represents a linear relationship between $(\sigma_{xx} - \sigma_{yy})$ and $(r)_{\theta=0}$ with slope D and T-stress being the y-intercept of a linear fit of normal stress difference data.

According to the different methods used in the literature confirm that the T stress is a constant value at the crack tip, or nearly [22, 24]. By contradiction, a new approach given by Hadj Meliani et al. [39], confirm that the T-stress is not constant for cracks and notches [40] and proposed method how take an average value using the volumetric method [25].

In this paper, we revise the Williams equation looking about the analytical solutions and we proposes numerical work using directly a single finite element (FE) analysis 2D by ANSYS V15.0 program [41]. For the computing method, the elastic T-stress efficiently and accurately by evaluating at the crack-tip. A Maleski method for computing T-stress was presented in mode I for CT specimen by modifying the threes methods [22-24].

2. Background

Several numerical and analytical methods were developed to calculate the elastic T-stresses [21-24]. Many researchers have provided T-stress solutions for 2D cracked bodies under uniform tension or bending loading conditions [13, 42-43]. Yang et al. [23] proposed the Stress Difference Method (SDM) to compute T-stress at crack tip. It incorporated the iterative single-domain dual-boundary element method and a tip-node rule to impose zero

displacement jump at the crack tip (Fig.1). The difference between σ_{11} and σ_{22} was demonstrated to evaluate T-stress [44].

Chao et al. [24] has proposed a simple method to calculate T-stress. T-stress is evaluated from stress distribution generally computed by finite element method and applying σ_{xx} in direction $\theta = 180^\circ$ (in the crack rear back direction) and define T-stress as the value of σ_{xx} in region where the value is constant [45].

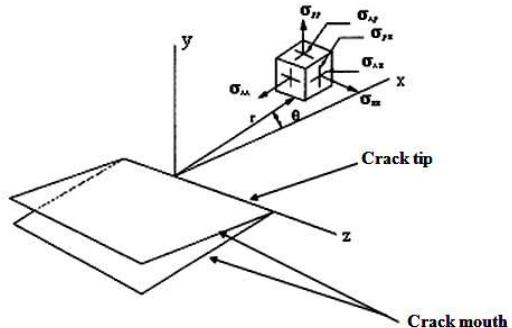


Fig.1. Stress Field in the vicinity of the crack.

Ayatollahi et Al [22] have determined T stress by using the displacement method in finite element and obtain then a stabilised T-stress distribution along ligament. In theory (1) should provide T within a reasonable distance from the crack tip. But in practice it is seen that FE results are not acceptable unless a large number of elements are used to simulate the crack tip zone, see for example Kardomateas et al. (1993) and Sherry et al. (1995). An improved method of obtaining the T-stress without recourse to much mesh refinement is to use the displacements along the crack faces. Due to traction free boundary conditions along the crack faces, Hooke's law can be written for small strains as [22]:

$$T = \sigma_{xx} = E' \varepsilon_{xx} = E' \frac{dU_{xx}}{dx} \quad (1)$$

Where ε_{xx} and U_{xx} are the strain and displacement respectively parallel-to-the-crack and E' is defined as:

$$E' = E \text{ Plane Stress} \quad E' = \frac{E}{1-\nu^2} \text{ Plane Strain}$$

Where E is Young's modulus and ν is Poisson's ratio. Ayatollahi [22] obtained that the displacement method gives more reliable results with less mesh refinement for T in comparison with that of the stress method [22]. Maleski et al. [21] has determined a modified stress difference method to calculate the T-stress. The normal stress difference ahead of the crack is given by:

$$[\sigma_{xx} - \sigma_{yy}]_{\theta=0} = T_0 + D(r/B) \quad (2)$$

where D is the higher-order coefficient associated with the r^1 term in the asymptotic expansion for $(\sigma_{xx} - \sigma_{yy})$ and T_0 . The above represents a linear relationship between $(\sigma_{xx} - \sigma_{yy})$ and $(r)_{\theta=0}$ with slope D and T-stress being the y-intercept of a linear fit of normal stress difference data, this is demonstrated in Figure 2.

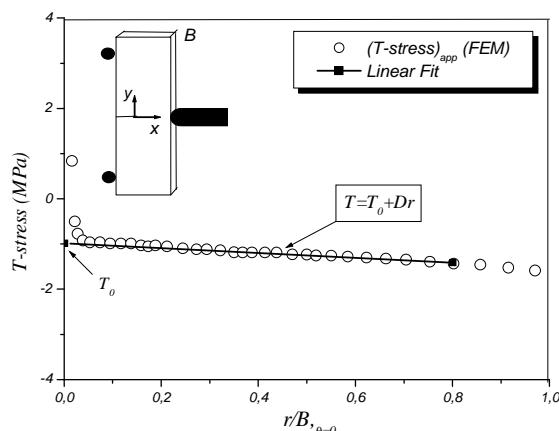


Fig.2. Typical linear regression of finite element data to determine T-stress using the modified stress difference method of Maleski [21].

Hadj Meliani et al. [25] has new approach for the T-stress estimation for specimens with a U-notch, used the most simple method has been employed to calculate the T-stress in a notched body such as SDM. The T-stress for the notch has been evaluated by experimental and numerical methods. Hadj Meliani et al. [25] has compared the Volumetric Method (VM)

in a notched body by stress difference method because it is the most simple and widely used and then allows comparison of our results.

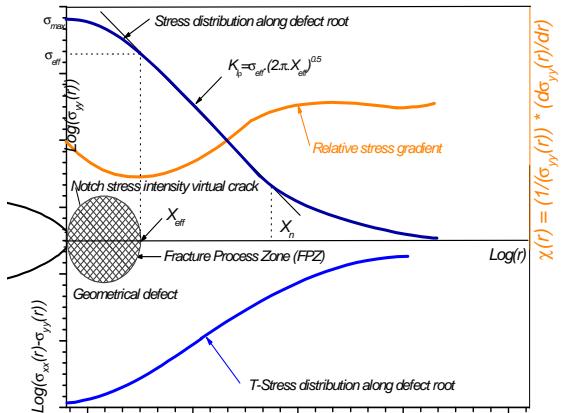


Fig.3. Average value of T-stress by the volumetric method [25].

Hadj Meliani. et al. [46] has the concept of the T -stress as a constraint factor has been extended to notch tip stress distribution. The effective T -stress (T_{ef}) has been estimated as the average value of the T -stress in the region ahead of the notch tip at the effective distance. The notch fracture toughness $K_{p,c}$ and the critical value of $T_{ef,c}$ have been determined using the volumetric method. The experimental method was used to validate the effective T -stress obtained by finite element method for different specimen geometries. Notch fracture toughness transferability has been proposed as a $K_{p,c}-T_{ef,c}$ curve and established from the tests of four specimen types (CT, SENT, DCB and RT) made from X52 pipe steel. A material failure curve $K_{p,c}=f(T_{ef,c})$ is established for the specimens under consideration. Fracture conditions are then given by the intersection of the material failure curve and fracture driving force curve for gas pipes with the surface notch. Pluvinage et al. [40] has a review of the influence of T-stress on the crack path and out-of-plane constraint and on the influence of thickness on fracture toughness.

3. Results and Discussions

3.1 Analytic study

The in-plane linear elastic stresses around the crack-tip. The stresses for each of the fields can be written as an eigen series expansion (Williams, 1957 [3]). Near the tip of the crack, where the higher order terms of the series expansion are negligible, stresses [22], can given by:

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + T + (O\sqrt{r}) \quad (3)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + (O\sqrt{r}) \quad (4)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + (O\sqrt{r}) \quad (5)$$

where the subscripts x, y and z suggests a local Cartesian co-coordinate system formed by the plane normal to the crack front and the plane tangential to the crack front point; r and θ are the local polar co-ordinates, K_I is the stress intensity factor for mode I. The T in Eq.(3) is the elastic T-stress, representing a tensile/compressive acting parallel to the cracked plane.

By literature reviews, many authors [22,25,39,40] have been found for T-Stress calculation can be expressed by the following equations (Table.1):

Table.1. T-stress values according to measurement direction [25].

θ	$\theta=0$	$\theta=\pm\pi$	$\theta=\pm\frac{\pi}{3}$	$\theta=\pm\frac{\pi}{2}$	$\theta=\pm\frac{2\pi}{3}$
T	$\sigma_{xx} - \sigma_{yy}$	σ_{xx}	$\sigma_{xx} - \left(\frac{1}{3}\right)\sigma_{yy}$	$\sigma_{xx} - \left(\frac{1}{3}\right)\sigma_{yy}$	$\sigma_{xx} - \sigma_{yy}$

This results in table 1, is given by the following steps, based on negligent of the higher order terms of the equation (4), to get the following equations (6):

$$\frac{\sigma_{yy}}{f_{yy}(\theta)} = \frac{K_I}{\sqrt{2\pi r}} \quad (6)$$

By substituting eq (6) in eq (3) to give the equation:

$$T = \sigma_{xx} - g(\theta)\sigma_{yy} \quad (7)$$

$$\text{where: } g(\theta) = \frac{f_{xx}(\theta)}{f_{yy}(\theta)}$$

The Function of $g(\theta)$ is plotted in the Figure 5 for different orientations; for $\theta=\pm\pi$ the function of $g(\theta)$ tend to $+\infty$. Analytically, the T-stress is not calculated in this angle.

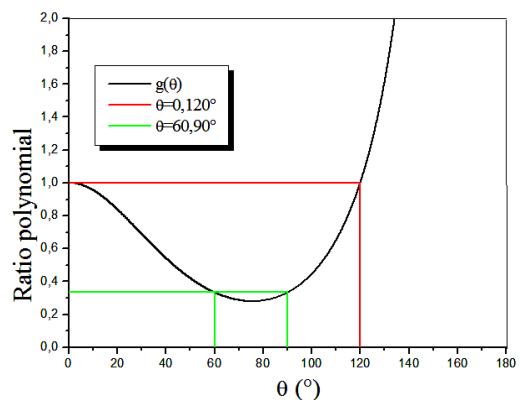


Fig.5. Distribution of $g(\theta)$ polynomial.

In other hand, we propose another analytical method to calculate the T-stress parameters. By the Subtraction depend on the deference of eq (3) and eq (4), describing in the equation (8), is presented in the Figure 6 for different angle. Table 2 recapitulates to T-stress evolution on the presence of the different angle.

$$T = \sigma_{xx} - \sigma_{yy} + \frac{K_I}{\sqrt{2\pi r}} h(\theta) \quad (8)$$

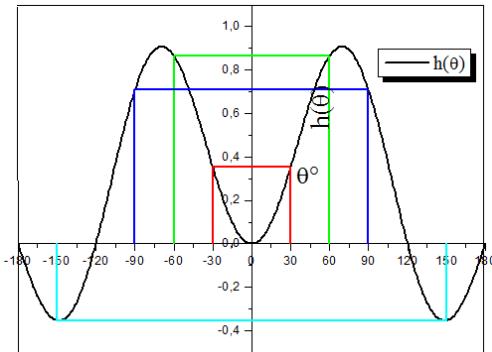
Fig.6. Distribution of $h(\theta)$ polynomial.

Table.2. Example of the recapitulative of the T-stress result for different angular positions around the crack tip.

θ	$\theta=0$	$\theta=\pm\pi$	$\theta=\pm\frac{\pi}{3}$	$\theta=\pm\frac{\pi}{2}$	$\theta=\pm\frac{2\pi}{3}$
T	$\sigma_{xx} - \sigma_{yy}$	$\sigma_{xx} - \sigma_{yy}$	$\sigma_{xx} - \sigma_{yy} + \frac{\sqrt{3}K_I}{2\sqrt{2\pi}r}$	$\sigma_{xx} - \sigma_{yy} + \frac{K_I}{2\sqrt{\pi}r}$	$\sigma_{xx} - \sigma_{yy}$

In this proposed analytical method, the results by using the new method of william's equation demonstration, given the same formulation of T-Stress at the angles $\theta = 0, 120^\circ$ and 180° . But it's not for another angles (see the Table 2). In other side the Chao method is a method based a stress deference method ($\sigma_{yy} \approx 0$). So, the Chao method described in the table is a single case of the Stress Difference Method.

The calculate of T-stress for in $\theta = 0^\circ, 120^\circ$ and 180° is based in the stress intensity factor value, this factor is according to the crack length so:

$$T = \sigma_{xx} - \sigma_{yy} + \frac{C(a, \theta)}{\sqrt{r}} \quad (9)$$

where: $C(a, \theta) = \frac{K_I}{\sqrt{2\pi}} h(\theta)$

3.2 Numerical Study

Finite element analyses using ANSYS V15.0 [41] were employed to determine T for mode I using the Stress Difference Method (SDM), Chao method (CM) and Displacement method (DM). For a CT specimen shown in Figure 7. The crack length a/W ratio variation at 0.2 to 0.4 (Table.2). We used an element PLANE183, this element is defined by 8 nodes or 6 nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element may be used as a plane element (plane stress, plane strain and generalized plane strain) or as an-axisymmetric element. This element has plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials. Initial state is supported. Various printout options are also available. We have used CT specimen with $W = 50.8$ mm (Fig.7), in two dimensional with plan stress. The used load is a failure load according to relative crack depth a/W of CT specimens (Table.2).

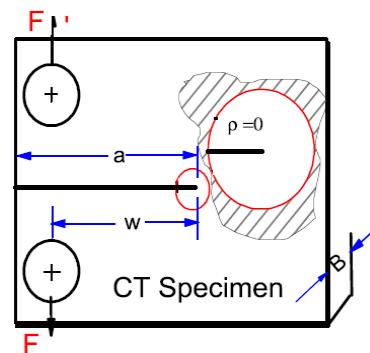


Fig.7. CT specimen with a crack [46].

Table.2. Failure load according to relative notch depth a/W of CT specimens [47].

a/W	0.2	0.3	0.4	0.5	0.6
F [N]	32.098	25.270	18.988	12.570	5.878

In figure 8 and figure 9 we present example of mesh and the distribution of Von Misses stress respectively, at crack near for a depth $a/W=0.4$.

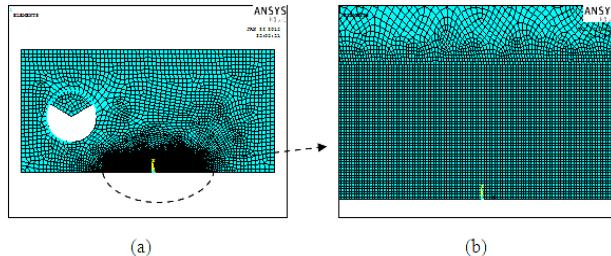


Fig.8. Typical meshes used for CT specimen and $a/W=0.4$.
 (a) View complete (b) Refined meshing in near the crack tip.

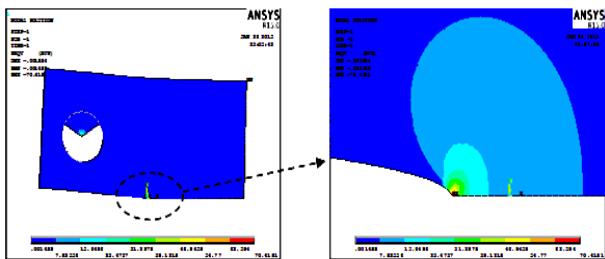
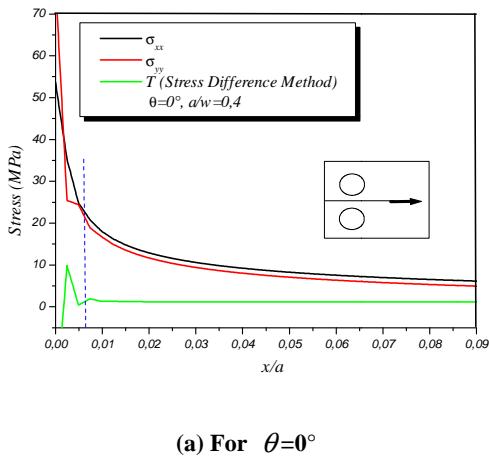
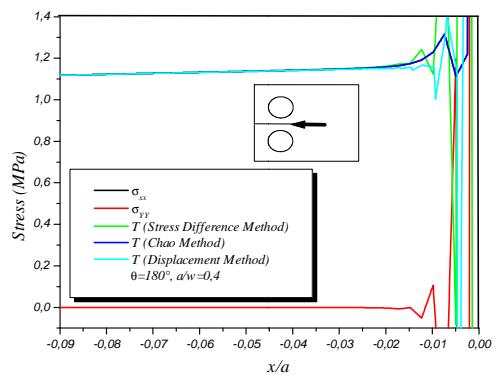


Fig.9. CT specimen (a) Evolution of stress Von Misses distribution for a depth ratio $a/W=0.4$ (b) Different contours of the stress Von Misses near the crack tip.

Figure 10 shows a graphic representation of the stresses σ_{xx} , σ_{yy} and T-stress distribution, relative to a depth $a/W=0.4$ and failure load 18,988 N for a angle $\theta=0^\circ$ and $\theta=180^\circ$.



(a) For $\theta=0^\circ$



(b) For $\theta=180^\circ$

Fig.10. Stress distribution σ_{xx} , σ_{yy} and T relative to a CT specimen for a depth $a/W=0.4$ and failure load 18,988 N

In figure 10.(a) we have the distribution of σ_{xx} and σ_{yy} they to lay a about constant distance, has to leave a $x/a \approx 0.006$, in this value we gives a constant stability of T-stress. In figure 10.(b) we have the three methods given a reliable result for a $\theta=180^\circ$ and we noticed that σ_{yy} tend towards 0.

3.2.1 Comparison of methods

Several numerical works used William's equation for obtaining the T-stress, for a CT specimen; the Stress Difference Method (SDM), Yang et al. [23], Chao method [24] and the Displacement Method of Ayatollahi et al. [22]. The calculate the T-stress by this methods is based for T-Stress describes a constant stress parallel to the crack direction. Chao [24] indicate that near crack tip, there exists nearly constant σ_{xx} region, that is $x/a = -0.001$ to -0.01 and this σ_{xx} value is chosen as the T-stress.

In this part we have a comparing between the tree method (SDM, CM and DM) in two directions ($\theta=0^\circ$ and $\theta=180^\circ$), for $a/W=0.4$ (Fig.12).

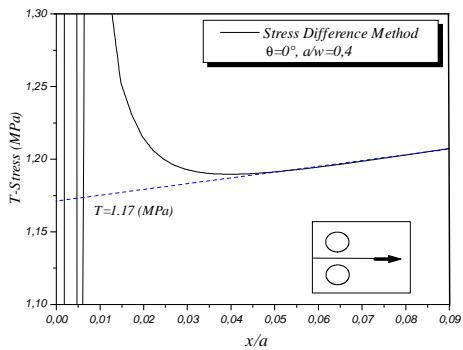
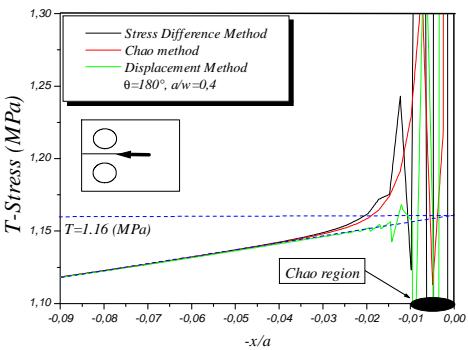
(a) For $\theta=0^\circ$ and by SDM(b) For $\theta=180^\circ$ and by three method

Fig.11. T-Stress distribution relative to a CT specimen for a depth $a/W=0.4$ and failure load 18,988 N.

In Figure 11 we have the three methods given a same path. But T-Stress describes a not constant stress parallel to the crack direction, even in the region of Chao we don't give a stabilization of T-stress describes. It should be noted that the present results of the effective T-stress estimation is consistent with the results obtained by the method proposed by Maleski et al. [21]. It was suggested that the T-stress can be represented by the following relationship (2). This method is presented by blue line. The compression between the T-stress value at $\theta=0^\circ$ and $\theta=180^\circ$ for $a/W=0.4$ is give a good result.

3.2.2 Influence of crack length a/W

In this part we have a comparing between the tree method (SDM, CM and DM) in two directions ($\theta=0^\circ$ and $\theta=180^\circ$), for $a/W=0.4$. In Figure 12 it present example of T-stress distribution obtained by difference method used relative to a depth variation at $a/w=0.2$ to 0.6 and failure load for a angle $\theta=0^\circ$ and $\theta=180^\circ$.

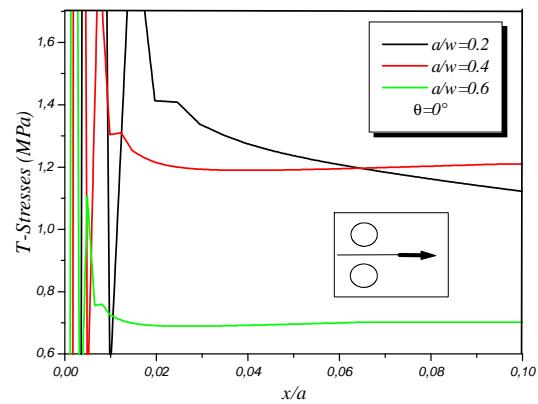
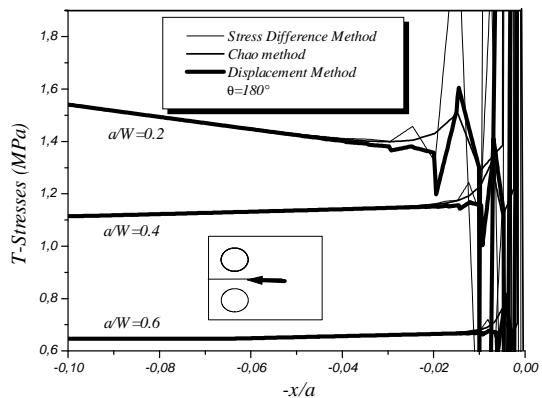
(a) For $\theta=0^\circ$ and by SDM.(b) For $\theta=180^\circ$ and by three method.

Fig.12. Stress distribution T relative to a CT specimen for a difference method used and value of depth a/W .

In Table.4 and figure 13 collect the result of the T-stress obtained by Finite Elements Analysis by Maleski method, for a/W variation at 0.2 to 0.6 in $\theta=0^\circ$ and $\theta=180^\circ$.

Table.4. Result of T-stress variation at two directions
($\theta=0^\circ$ and $\theta=180^\circ$).

a/W	T-Stress (MPa)	
	$\theta=0^\circ$	$\theta=180^\circ$
0.20	1.35	1.30
0.225	1.20	1.18
0.25	1.11	1.10
0.275	1.08	1.07
0.30	1.07	1.07
0.35	1.12	1.12
0.40	1.17	1.16
0.50	1.06	1.06
0.60	0.675	0.67

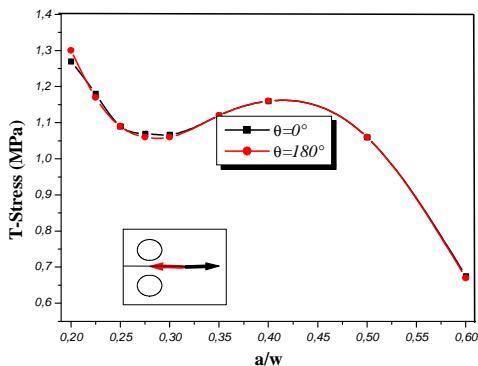


Fig.13. The value of T-stress variation with two directions.

In figure 14 it present the variation of the tangent of Maleski (2) for a/w variation at 0.2 to 0.6 in $\theta=0^\circ$ and $\theta=180^\circ$.

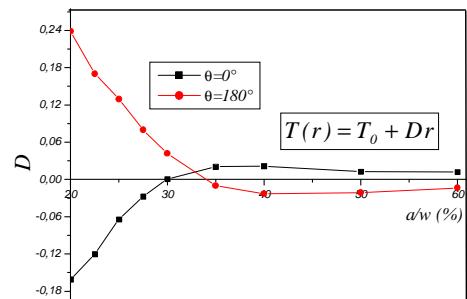


Fig.14. Tangential of Maleski distribution relative to a CT specimen for a depth $a/W=0.2$ to 0.6.

In figure 14 we have the T-Stress describes a constant stress parallel to the crack direction at $a/W=0.3$ for $\theta=0^\circ$, and $a/W=0.33$ for $\theta=180^\circ$.

4. Conclusions

The Williams's type solution has been employed to analyse the stress distribution ahead of the crack tip. It was observed that the T-stress values are positive (tension stress) for a CT specimen in the interval $a/W=0.2$ to 0.6, and shown that the T-stress is not constant along ligament ($\theta=0^\circ$) and crack mouth ($\theta=180^\circ$) ahead of the crack tip for CT specimen. It was also found that the non-singular terms are not negligible for a crack as the distance from the crack tip increases. To avoid this difficulty, it suggested to use the Maleski method. The extrapolation method for calculation of T-stress value, in two direction ($\theta=0^\circ$ and $\theta=180^\circ$) ahead of the crack tip. Thus, the concept of the T-stress in the case of the crack stress distribution has been extended to the crack stress distribution.

This result improve that the T-Stress value is not constant for a crack depth ratio variation, and with a distribution not really established as in theory. The distribution of σ_{yy} is negligible for a $\theta=180^\circ$, in this angle the SDM and Chow method give approximately

the same result, this result is different at the result of AYATOLAH [22]. The Maleski method provide a good result for a $\theta=0^\circ$ and $\theta=180^\circ$.

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