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Auteur correspondant : benyahi.karim@yahoo.fr

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Reliability analysis of three-dimensional reinforced concrete structures by the Monte-Carlo method

BENYAHI Karim[a], BOUAFIA Youcef [a], BOUCHAFADjamal[a]

[a]*Laboratoire de Modélisation des Matériaux et Structures (LAMOMS), Université Mouloud Mammeri de Tizi-Ouzou, BP 17 RP 15 000 Tizi-Ouzou, Algérie.*

Abstract

The objective of this study is to analyse the reliability of three-dimensional reinforced concrete structures in which the non-linear mechanical and geometric nonlinearity are take into account. For this purpose, we have developed a program in FORTRAN to simulate the direct Monte Carlo method coupled to a non-linear calculation for evaluating the failure probability of three-dimensional structures. The events of the environment (field strengths, actions) and materials behaviour are simulated by random and deterministic variables. A first validation illustrate the mechanical modeling approach; by comparing the results of present calculation with experimental results or analytical solutions. The reliability-model is checking; the simulation results of the direct method of Monte Carlo are compared with those of Hasofer-Lind.

Keywords : Modeling ; nonlinearity ; reliability ; failure probability ; Monte Carlo

Nomenclature

P :	Applied vertical load
H :	Applied horizontal load
L :	Range
h :	Height
v :	Poisson ratio
f_{cj} :	Concrete compressive strength
f_{tj} :	Concrete tensile strength
E_b :	Concrete young modulus
σ :	Stress
σ_e :	Yielding stress of the reinforcement
E_a :	Steel young modulus
k_b :	Sargin law parameter
k'_b :	Sargin law parameter
ε :	Strain
ε_u :	Ultimate strain reinforcement
ε_{cu} :	Ultimate concrete strain
ε_{ct} :	Concrete cracking strain
ε_{rt} :	Ultimate concrete tensile strain
P_f :	Failure probability
β :	Reliability index
$G(X1, X2)$:	Limit state function
δ :	Deflection

1. Introduction

The safety of a mechanical system is provided by a safety coefficient: the ratio of a load variable and a resistance variable are established by a deterministic approach. For a complex structure, these efforts are not well known, and its resistance is uncertain, then; there is always a risk of seeing the ruined structure. For this, the probabilistic approach for assessing the risk by reliability analysis methods of mechanical systems is developed in recent years [1]. This study aims to develop high-performance digital tools in terms of computation time and accuracy for estimating the reliability of column and beam structures in reinforced concrete.

2. Nonlinear behavior of structures

The assumption of linear behavior of structures has a limited range of validity. Indeed, model the behavior of real materials calls made to a non-linear stress-strain relationship. Also, structures are becoming lighter and tend to be subjected to large excitations which lead to large displacements and deformations. Thus, there are various sources of possible non-linearity in the structures:

- The material nonlinearities in small displacements and small or large deformations, the stress-strain relationship is nonlinear. They may come from the constitutive law, the cracking of the material behavior of concrete stretched between two cracks.

- The geometric nonlinearities in large displacements and small deformations, the displacement-strain relationship may be nonlinear. They may develop instabilities slender shape with movable parts or nodes slender structure.

The beginning of the nonlinear analysis goes back to 1968 when NILSON [in 2] develops a non-linear analysis of reinforced concrete structures using the finite element method. The geometric nonlinearity in two dimensional (2D) may be considered accurately through additive and commutative planar rotations. Discussed in the context of this work using non-linear calculation software using a model that takes into account the combined effect of the planar bending and shear in nonlinear elasticity [2-7]. This model is applied to reinforced concrete beams.

3. Reliability of structures

In the words of A.M. Freudenthall: the reliability theory is to place the concept of the safety of structures in the realm of physical reality where there are no absolutes, where knowledge is not perfect.

Thus, the reliability engineer approach consists to introduce a quantitative measure of default risk using a probabilistic approach to modeling uncertainties and contingencies. Indeed, the problem of reliability of structures is based on two fundamental aspects: a physical knowledge of the mechanical phenomena of the various failure mechanisms, on the one hand, and statistical knowledge of the parameters of the mechanical model, on the other hand (Figure 1).

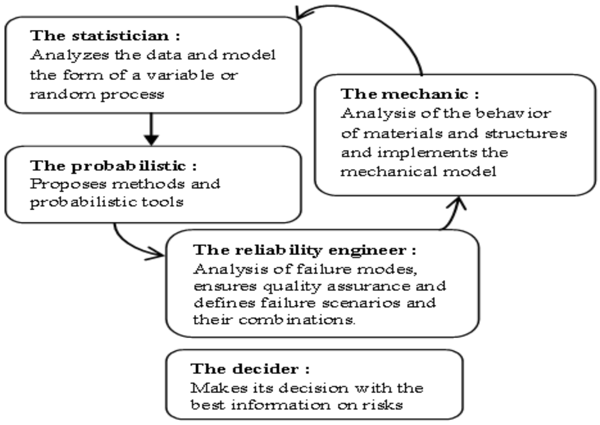

Fig. 1. Actor-mechanical reliability-coupling [1].

3.1. Directly Monte Carlo simulation

This method is based on a succession of random samples in the space of physical parameters. These prints are made according to the law of spatially variable physical variables.

The limit state function between the parameter space in a secure area and an area of failure can be written as follows:

$$G(\delta, P) = \delta_{max} - \delta_{calcul} \quad (1)$$

G(δ,P)=δ_max-δ_calcul

$G > 0$: defines the safe area;
$G \leq 0$: defines the failure area and;
$G = 0$: defines the limit state.

3.2. Methodology

This is to make a large number of random draws k (that is to say, generate realizations of random variables according to their joint probability density). The steps are:

- Calculate the value of the limit state function for each draw;

- Test the function of limit state for each draw:

  - If $G \leq 0$: There is failure, increment the counter of failed case based on the total number of performed achievements;

  - If $G > 0$: There are not failure, no incrementing the counter (failure indicator);

- Repeat the process from 1 to k until a sufficient number of draws is reached;

- Calculate the probability of failure:

$$\lim_{n\to\infty} \frac{\text{number of events where} G \leq 0}{\text{total number of simulated events}} \quad (2)$$

4. Validation of the method adopted

4.1. Study of the beam OG3

The non-linear calculation program used is three-dimensional structures computation software for reinforced and prestressed concrete [2 to 7], it allows to evaluate the resistance of column-beam reinforced concrete structures to the break in nonlinear elasticity under instant load. The beam studied in this work is illustrated in Figures 2, 3 and 4, the data reliabilities and mechanicals characteristics of the beam OG3 [in 7] are given in Table 1.

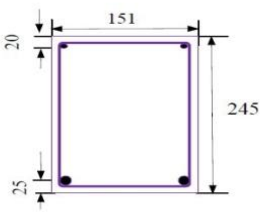

Fig. 2. The beam OG3.

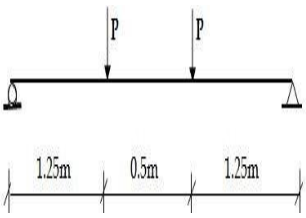

Fig.3. Section A-A of the beam

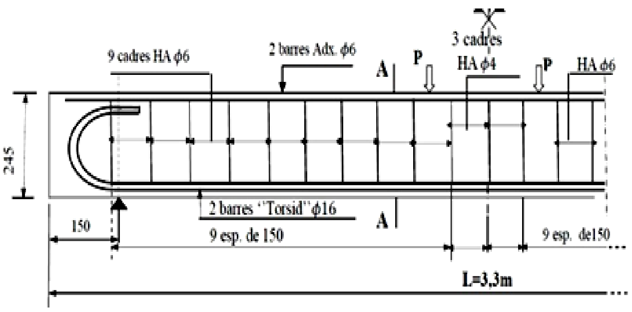

Fig. 4. Reinforcement of beam OG3.

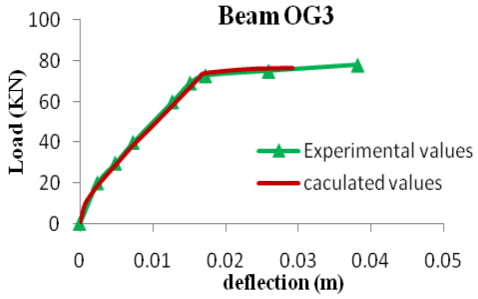

Fig. 5. Evolution of the vertical arrow in function of load P.

Table 1
Data reliabilities and Mechanicals characteristics of the beam OG3

Variable	Unit	Probability law	Average	Standard deviation
P	$[KN]$	Deterministic	Variable	–
L	[m]	Deterministic	3	–
v	–	Deterministic	0.2	–
f_{cj}	[MPA]	Normal	52.50	5.25
f_{tj}	[MPA]	Normal	3.35	0.335
E_b	[MPA]	Normal	$3.99.10^4$	$3.99.10^2$
σ_e	[MPA]	Normal	575	57.50
E_a	[MPA]	Normal	$2{,}05.10^5$	$2{,}05.10^3$
k_b	–	Deterministic	1.29	–
k'_b	–	Deterministic	0.29	–
ε_{cu}	–	Normal	0.0035	0.00035
ε_u	–	Normal	0.05	0.005

- *Evaluation of the arrow in load (P)*

The simulation is intended to evaluate the vertical force (P) according to the vertical arrow in the middle of the beam until its breaks. For this, we introduce the necessary data OG3 beam (Table 1) in the computer program developed following the standard Fortran 90.Behavior laws used for digital simulations are Sargin law for the behavior compressive concrete and Grelat model for the tense concrete. As for the steel behavior laws, we took those work-hardened steels and elastoplastic. The choice of one of these laws depend on the steels used in the experimental tests. The evolution of the load versus displacement is given in Figure 5.

The simulation shows a good estimate of the failure load and behavior is simulated up to the ruin of the beam. For against, the ultimate displacement is somewhat underestimated relative to the displacement obtained experimentally, this is probably due to the method (tangent) of used numerical resolution.

- *Reliability analysis*

This step is to apply the adopted resolution method (Monte Carlo) to calculate the probability of failure. In what follows, one proceeds to the evaluation of the probability of failure of the beam using two methods: the level of method II (HL-RF) [1, 8] to calculate the probability of failure and the level of method III (direct Monte Carlo simulation) to verify the results from the methods FORM / SORM.

- *The level of method II (Hasofer-lind-Rackwitz-Fiessler)*

Given the complexity of the finite element model, it is difficult to conduct the study by a direct coupling between the nonlinear calculation program and reliability program, then; it becomes necessary to construct an analytical response surface.

Random input variables are the parameters describing the nonlinear behavior of the material (concrete elastic Young modulus; maximum concrete compressive stress; maximum concrete tensile stress; steel elastic limit;steel Young's modulus) and law of random distributions (given in table 1). The other parameters such as the length of the beam are considered as deterministic.

The procedure is applied to estimating random variables probability distributions (see Figures 6 and 7 and Table 2). It included in this study in order to approximate the statistical law (real) and also to introduce them into the calculation program to the reliability of the structures [8].

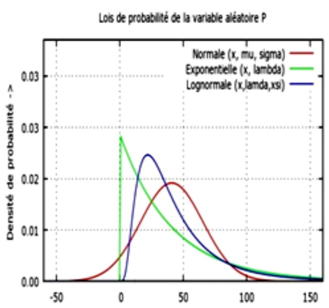

Fig. 6. Probability of the random variable laws P.

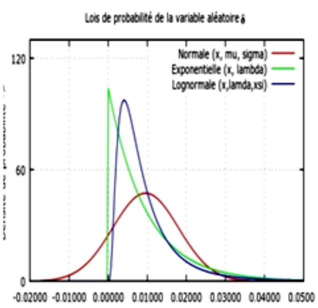

Fig. 7. Probability of the random variable laws δ.

Table 2

Parameter laws of random variables for limit state

X vector	X1	X2
Random variables	*P*	*δ*
Distribution law	Exponential	Lognormal
Average $\boldsymbol{\mu}$	40.979	0.0095
Standard deviation $\boldsymbol{\sigma}$	24.499	0.0084
Coefficient of variation $\boldsymbol{\nu}$	0.597	0.878

After analysis by coupling mechanical reliability engineer, the HL-RF method allowed us to obtain the results given in Table 3.

Table 3

Results of mechanical reliability analysis

Reliability index	1.2930
Probability of failure	0.0985 (9.853%)
Reliability	0.90147 (90.14%)
Point centered design reduces space (U1, U2)	(0.3901, -0.2403)
Point of physical space design (X1, X2)	(25.311, 0.0044)

The reliability index is$\beta = 1.29306$; which corresponds to a probability of failure of the beam (OG3) estimated to $P_f = \Phi(-\beta) = 9.853\%$, and resulting in a reliability of 90.147%. As; to the most probable failurepoint, in physical space, correspond to the load 25.311 KN and the displacement 0.0044 m.

- *Directly Monte Carlo*

After Monte Carlo simulation analysis, the probability of failure is: $P_f = 11.90476\ \%$, then the structure is reliable to $88.09524\ \%$. It is found that the results obtained after analysis by Monte Carlo simulation ($P_f = 11.90476\ \%$) are close to those of the method of HL-RF [8] ($P_f = \Phi(-\beta) = 9.853\%$). These results highlight the need to deepen the knowledge of material properties to narrow the distribution parameters and take into account any correlations between random variables.

4.2. Study of a bi-articular Cranstan frame

The frame studied in this work is shown in figure 8.

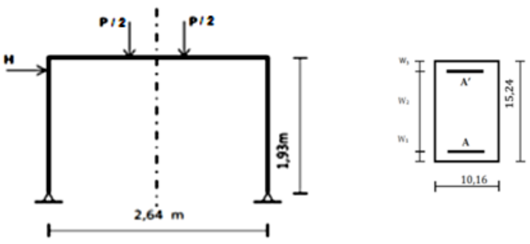

Fig. 8. Frame of W.B. Cranston [2].

- *Evaluation of the arrow in the load (P)*

The simulation is intended to evaluate the vertical force (P) according to the vertical arrow in the middle of the beam until it breaks. For this, we introduce the necessary frame data (see Table 4).To take account of the confined concrete, we opted for the law proposed by Bouafia et al. [3] which shows the percentage of confinement ranging from 10-25‰.

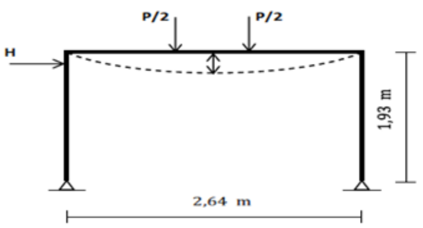

Fig. 9. Arrow studied.

As for the steel behavior laws, we took those work-hardened steels and elastic-plastic. The choice of one of these laws depends on the steels used in the experimental tests.

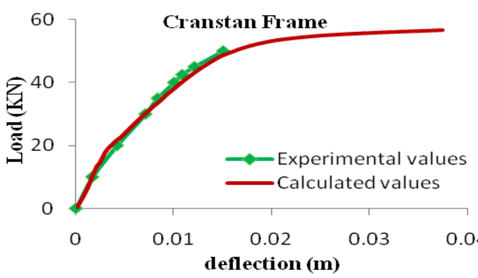

Fig. 10. Evolution of the vertical arrow according to the load P.

The curve, shown in Figure 10, approximates the experimental curve in phase of elastic behavior, but at the onset of cracking phenomena, it begins to deviate from the experimental curve. This discrepancy is due to inaccuracies in the actual values for materials (such as the original elastic modulus of concrete or steel of which the distribution is not completely specified in the Cranston reference).

Table 4

Data reliabilities and Mechanicals characteristics of the Frame Cranston

Variable	Unit	Probability law	Average	Standard deviation
P	$[KN]$	Deterministic	Variable	–
H	$[KN]$	Deterministic	Variable	–
L	[m]	Deterministic	2,64	–
h	[m]	Deterministic	1,93	–
v	–	Deterministic	0,2	–
f_{cj}	[MPA]	Normal	34	3.4
f_{tj}	[MPA]	Normal	2.59	0.259
E_b	[MPA]	Normal	35656	3565.6
σ_e	[MPA]	Normal	278	27.8
E_a	[MPA]	Normal	$2{,}05.10^5$	$2{,}05.10^3$
k_b	–	Deterministic	2.15	–
k'_b	–	Deterministic	1.15	–
ε_{cu}	–	Normal	0.0035	0.00035
ε_u	–	Normal	0.02	0.002

- *Evaluation of the probability of failure of the frame*

In the following, we have proceeded to the evaluation of the probability of failure of the beam using two methods: the method of level II (HL-RF) [1, 8]and the method of level III (direct Monte Carlo simulation).

- *The method of level II (HL-RF)*

We yielded a probability of default that is equal to 5.15%, while the structure is 94.85 % reliable.

- *Directly Monte Carlo*

After Monte Carlo simulation analysis, the probability of failure is: $P_f = 7.11\%$, then the structure is reliable to 92.89 %.

5. Conclusions

The numerical results obtained on the frame of Cranston and OG3 beam are close to the experimental results, which allowed us to verify the validity of the non-linear calculation program of reinforced concrete structures.

The most used engineering methods are called approximate methods (FORM, SORM). Unfortunately, these methods do not have an error estimator. For this, the Monte Carlo simulation method was used jointly to check the approximate methods results. This method is extremely expensive in computing time. However, it remains one of the most robust.

The reliability-coupled model to the mechanical model applied to tested structures allowed us in this study to assess the probability of failure on the ruins of a nonlinear behavior structure subjected to random loads.

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