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Determination of stress intensity factors for a bi-material ring spicemen with curved cracks under compressive loading

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Abstract:

The performance in resistance of structures and the lightening in weight lead the researchers and engineers to use various materials in the same structure designed to be in service. The study of bi-materials took a dominating place these last years. In this paper, various numerical analyses using the finite element method have been carried out to calculate the stress intensity factors at the tips of curved cracks in bi-material ring. For the case of homogeneous material with a curved crack, the obtained results in term of stress intensity factors gives a good agreement with the analytical expression for infinite plate with curved crack.

Keywords: Stress intensity factors, Bi-material, Interface, Circular crack

1. Introduction

Currently, the bi-materials take an increasingly consequent place because of their properties. However, the study of bi-materials is complicated because of the geometrical and the material discontinuities, thus their behavior in rupture requires a particular study.The understanding of the failure mechanisms is very important, as much as the estimation of fracture parameters at the tip of the crack approaching an interface. The elasticity solutions of interface crack problems had been determinate for the first time in the late 1950's by Williams (1959) [1]. He solved the interface crack problem and discovered complex singularities and rapid oscillating stresses near the crack tip, also he estimated the oscillatory region to be very small. In the mid 1960's, Erdogan [2], England [3], Rice and Sih [4] considered as the first researchers who came up with closed-form solutions to the interface problem, these solutions verified Williams' (1959) finding of complex singularities and rapidly oscillating stresses near the crack tip. In 1977, Comninou [5], presented a new theory of elasticity for interface cracks, he resolved the problems at the crack tip by studying the interface. Recently, with the scientific development and the appearance of the numerical methods, various research works have been carried out by different researchers by using the finite element method to solve the problems of the interfacial cracks in bi-material plates. B. Serier and al [6] used it to analyze the behaviour of an interface crack in bi-material specimen with a central hole. The same authors [7] used it again to study the effect of interaction between an interfacial main crack and a sub-interfacial micro-crack in bi-materials. Another work was presented by [8] using the same method to evaluate the interaction effect between a crack and an interface in a ceramic/metal bi-material. The stress intensity factors for the Brazilian disc with a short central crack were determinate by [9]. L. Marsavina, T. Sadowski [10], study the case of a kinked crack whose corner is on the interface of two bonded dissimilar ceramic materials and calculate the stress intensity factors at the tips of the crack. A new method for obtaining the mixed-mode stress intensity factors for a bi-material interface crack in the infinite strip configuration and in the case where both phases are fully anisotropic was presented by [11].

The singular stress field near the bi-material notch tip was investigated by [12]. Tamine et al. [13] developed the effects of mechanical materials properties, position and orientation between crack and the bi-material interface, theirs results show that the energy release rate of the interface crack is influenced considerably by these parameters. A solution was presented for the problem of a finite length crack branching off the interface between two bonded dissimilar isotropic materials by [14]. Results are presented in terms of the ratio of the energy release rate of a branched interface crack to the energy release rate of a straight interface crack with the same total length. The special sub-region boundary element method [15] is used to obtain the stress and displacement distributions on the bi-material boundaries.

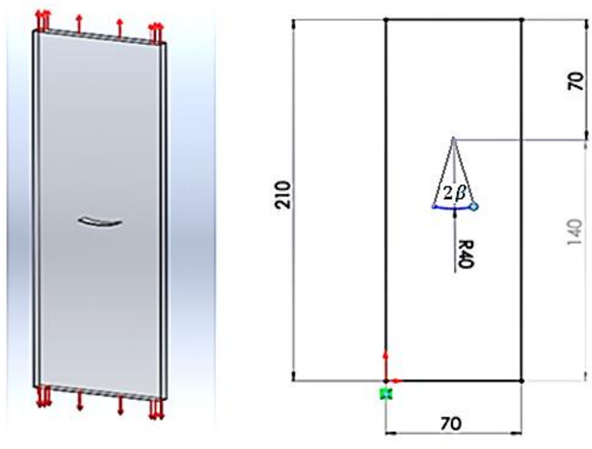
Experimental methods based on RGB (Red Green Blue) photo-elasticity has been carried out by Cirello and Zuccarelo [16] to analyse the propagation of cracks perpendicular to interface of bi-material plates. Another experimental method based on an improved digital phase-shifting photo-elasticity method has been used by [17] to determine the whole-field shear stress. In addition, the cracks can be presented in these materials not only with rectilinear forms but also with curved forms but the mechanical behavior of these forms has important implications even for the case of homogeneous materials. In this paper, we are being interested in this last type of cracks. The two dimensional problem of an open circular arc crack in an infinite isotropic, homogeneous, elastic material subjected to a uniaxial tension at infinity was first solved by Muskhelishvili in 1953[18].This solution was largely applied to study the behavior of curved cracks. In 1966, Perlman and Sih [19] have studies the case of circular arc cracks in bi-material plats under bending the study deals with the problem of the inflection of thin section inside contains a circular insertion of another material with lines of discontinuity along the bond which are considered as circular crack. In 1985, a method was proposed by Rice [20] for the elastic field of a crack with a front perturbed from some reference shape, to solve the elasticity problems of somewhat circular planar tensile cracks under arbitrary load distributions. The method is based on a known solution for the stress intensity factor along a circular crack due to a pair of wedge-opening point forces on its surfaces. In the latest years much research concerning the case of be circular cracks were published; we can mention here the work of M. Gomez and al [21] and the works of Y Y. Chen [22-24]. In 2011, Elizabeth Ritz and David D. Pollard [25] studied the solution of Muskhelishvili and defined with precision when this solution does not apply. They explain how and why the curved cracks are closed under a load range condition and calculate the stress intensity factors for circular cracks where the crack is partially closed.

2. Numerical determination of the stress intensity factors at a crack tip

The aim of this study is to calculate the stress intensity factors for a curved crack in a bi-material ring under a compressive load. In order to valid our numerical resolution with finite elements; we have studied the case of homogeneous material with curved crack. The results obtained in terms of stress intensity factor are compared with those expressed analytically by [26].

2.1. Curved crack in an infinite homogenous plate specimen

Let's consider a thin plate with curved crack, the dimensions of the plate are ($3L$ x $2L$) (Fig.2.2 b) made up of only one material. The curved crack has a constant radius R and a variable angle 2β. This structure was loaded under a uniform uniaxial tensile stress, perpendicular to the curved crack.The material plate is a polycarbonate (PSM-1) with mechanical properties (E = 2 300 MPa and υ=0,36)



(a) (b)

(a) Condition loads; (b). Dimension of the plate

Fig 1: geometrical model of bi-material plate with curved crack:

Numerical model carried out by using ANSYS software finite element have allowed us to determine the stress intensity factors from displacement field. The solution of the problem has been calculated by using 8 node quadratic elements in plane stress. The typical mesh model of the plate is presenting in fig 2.2

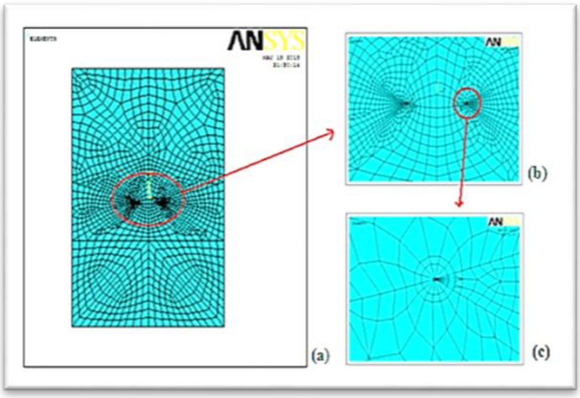


Fig 2: Meshed curved crack plate (a), with the meshing at a vicinity of rack (b) and (c)

For this problem, the reference solution for the stress intensity factor is provided by [26]. Numerical results are compared with results established by [26].

$$K_I = \frac{\sigma}{2}\sqrt{R\sin\beta}\left[\frac{\left(1-\sin^2\frac{\beta}{2}\cos^2\frac{\beta}{2}\right)\cos\frac{\beta}{2}}{1+\sin^2\frac{\beta}{2}} + \cos\frac{3\beta}{2}\right] \qquad (1)$$

$$K_{II} = \frac{\sigma}{2}\sqrt{R\sin\beta}\left[\frac{\left(1-\sin^2\frac{\beta}{2}\cos^2\frac{\beta}{2}\right)\sin\frac{\beta}{2}}{1+\sin^2\frac{\beta}{2}} + \sin\frac{3\beta}{2}\right] \qquad (2)$$

The numerical and analytical resolutions giving the stress intensity factors for the plate containing a curved crack subjected to traction are given in figure 3

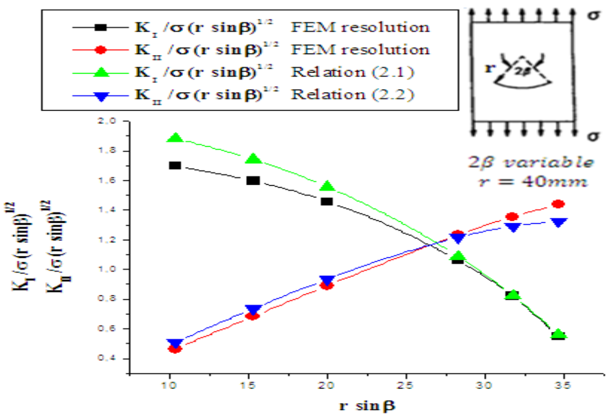


Fig 3: Evolution of the normalized stress intensity factors in mode I (opening mode) and mode II (plane shearing mode) according to the opening crack angle

The results obtained by the finite element method are in good agreement with those obtained from the analytical method [26]. The medium error is less than 5.9%.

2.2 Modeling and simulation for a circular crack in a bi-material ring

In the following simulation, we try to determine the variations of the stress intensity factors at the curved crack tips in a bi-material ring charged in compression. We consider bi-material cracked specimens (aluminum/PSM-1) showing in Fig (2.4). Different crack sizes and different crack positions have been chosen;

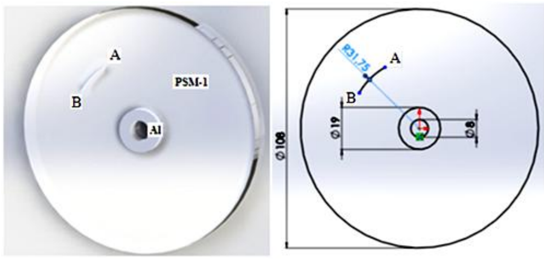


Fig.4: the bi-materials ring with a curved crack

The geometrical data of the structure are presented in the following table.

Table 1

Geometrical data of the structure

External diameter	$D_{ext} = 108$ mm
Diameter of interface	$D_{intetface} = 19$ mm
Internal diameter	$D_{int} = 8$ mm
Thickness	t = 5,94 mm

The mechanical properties of materials are represented in table 2.

Tab 2

Elastic constants of the materials

	Young Modulus (*MPa*)	Poisson ratio υ
PSM-1	2 300	0.36
Aluminium	69 000	0.30

2.1.1. Determination of the stress intensity factors according to the opening angle of the curved crack

The influence of the curved crack size on the stress intensity factor with the presence of the interface is determinate in this case. The curved crack is located in PSM-1 with a constant radial position r=31.75 *mm*. The different values of the opening angle 2β are: 10°, 20°, 30°, 45° 60° and 75°.The mechanical loading is in the form of diametrical compression corresponding to an opening loading angleω=70°. The values of the normalized stress intensity factors obtained by the finite element method with code ANSYS are given in figure 5

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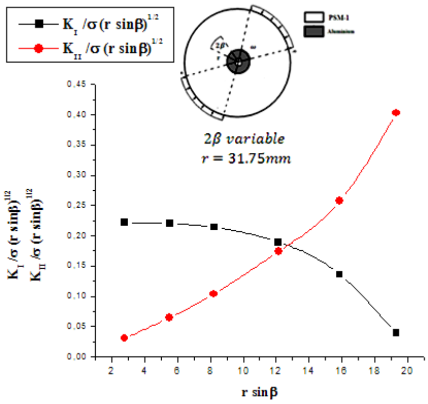


Fig 5: Evolution of the normalized stress intensity factors in mode I (opening mode) and mode II (plane shearing mode) according to the opening crack angle for the ring specimen

It must be clarified that the solution presented in figure 5 is only valid for the specific combination (β, ω) ensuring the not-closing of the curved crack, in which way, we obtain an acceptable physically results. When the opening angle of the crack 2β is equal or higher than 75° ($2\beta \geq 75°$), the lips of the crack are superimposed and the calculation of the SIF for these cases are not possible. According to figure 5, we notice also the presence of the mixed mode defined by the angle phase ψ as $\psi = arctan\,(K_{II} \,/\, K_I)$, and it can be measured by the calculation of the angle ψ . The results obtained show two values; a minimal value of $\psi = 7.87°$ for $2\beta = 10°$, and a maximum value of $\psi = 84.48°$ for $2\beta = 75°$.

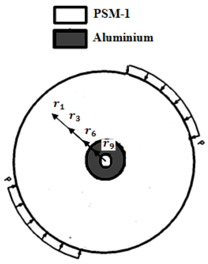
2.1.2. Determination of stress intensity factors according to the variation of the radial coordinates of the curved crack

This part of work is devoted to study the variation of the radial location of curved crack and his influence on the evolution of the stress intensity factors. We consider the opening angle curved constant $2\beta = 30°$. The curved crack will be located according to its radial position either in the polycarbonate material or in the aluminum. Table 3 gives the values of the radial location of curved crack.

Tab 3

Radial location of curved crack along the bi-material ring

Radial variation of curved crack in the PSM-1 (mm)	
r_1	42.875
r_2	37.312
r_3	31.750
r_4	26.187
r_5	25.000
r_6	23.000
r_7	21.00
Radial variation of curved crack in the aluminum(mm)	
r_8	8.12
r_9	6.75



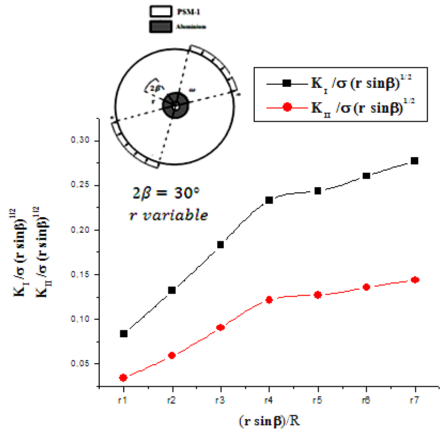


Fig. 6: Evolution of the normalized stress intensity factors according to the radial location of the curved crack.

The numerical results are plotted in Figures 6.From this figure, it is clear that the normalized stress intensity factors takes a maximum values close to the interface, where the opening angle of the curved crack 2β is constant. The obtained values increase when the radial locations decrease. When the crack is located in the most rigid part of the bi-material which is aluminum, the crack is closed under the effect of the pressure applied, and in this case, it is impossible to calculate the stress intensity factor. To decrease the effect of the difference in rigidity of materials, the bi-material (steel/aluminum) is studied and the results are given in figure 7.

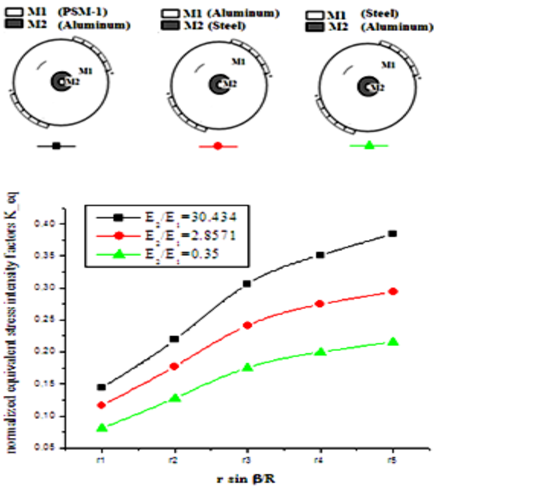


Fig 7: Evolution of the normalized equivalent stress intensity factors K_{eq} with the variation of the radius R for various reports of the Young Modulus$E$

The normalized equivalent stress intensity factor is given by the relation:

$$K_{eq} = \sqrt{\left(\frac{K_I}{\sigma\sqrt{r\sin\beta}}\right)^2 + \left(\frac{K_{II}}{\sigma\sqrt{r\sin\beta}}\right)^2} \quad (3)$$

The normalized equivalent stress intensity factors curves in figure 7 increase for all the values of radius R. These results prove that the increase of $K_{eq}$with the distance separating the curved crack from the interface (radius R) strongly depends on the ratio of the Young Modulus (E_1/E_2) of both materials. More rigidity of the material where the crack is located reduced more K_{eq} increases.

2.2.3 Determination of the stress intensity factors according to the variation of the angular coordinate of the curved crack

In this part of the study we considered the variation of the angular coordinate giving the position of the crack with respect to the y axis of the ring. The radius R and the opening angle of curved crack 2β are assumed to be constant. The values of the angular coordinate α with respect to the Y axis are:15°, 20°, 25°, 30°, 35°, 40° and 45°.

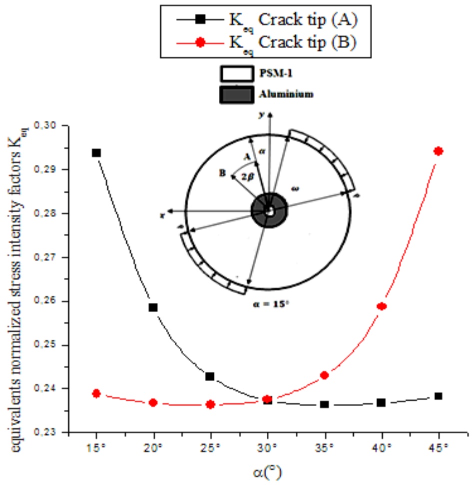


Fig 8: Evolution of the equivalent normalized stress intensity factors for the both crack tips A and B according to the angleα.

The values of the normalized equivalent stress intensity factors for the both crack tips A and B (fig. 2.8), strongly depends on the angular distance defined by the angle α which designates the position of the crack tips with respect to the loading area.At the crack tip (point A) in the right side of the ring ($\alpha = 15°$), K_{eq} is maximum. In the other side and for the same angle, K_{eq} for the crack tip (point B) is minimal. For the crack tip in the left side of the ring (point B) ($\alpha = 45°$), K_{eq} is maximum and for the same angle this value is minimal in the crack tip A. We also noticed that for $\alpha = 30°$, the both of crack tips (A and B points) are symmetrical with respect tothe loading axis and have the same values of stress intensity factors.

3. Conclusion

In this paper the finite element method has been used to determinate the stress intensity factor in a bi-material ring containing curved cracks solicited mechanically. Initially, the results obtained about the evolution of the stress intensity factors in the case of an isotropic homogeneous plate with a curved crack are validated with those calculated analytically. In the second part, an annular bi-material structure containing a curved crack was modeled. For all the cases presented in this paper, the curved crack remains was always parallel to the interface. The results obtained numerically by the finite element method in terms of stress intensity factor depend on several factors, and it appears that:

- The normalized stress intensity factors strongly depend on the size of the crack, more the opening angle of the crack 2β increase more the SIF's values increases.
- For the same size of the crack, the distance between the crack and the interface, influences directly on the normalized stress intensity factors. this last one increase with the reduction of the radial distance and take maximum values close to the interface
- The study of the variation of various ratio of the Young modulus for the same crack and the same radial distance shows that the equivalent normalized stress intensity factors are strongly depend on this ratio. Weaker is the rigidity of the material where is located the curved crack, more the increase of $K_{eq}$was remarkable.
- The values of the SIF's for the both tips of the curved cracks were strongly depending on the angular position of the crack. More the crack's tip approach towards the loading area, more the equivalent normalized stress intensity factor increases.
- It is noted that the mixed mode was present in the homogeneous and heterogeneous structure in the presence of the curved crack and that independently of its position and the presence of the interface.

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