SHOW WAYS IN WHICH THE USE OF DIAGRAMS, FIGURES OR GRAPHS CAN AID THE UNDERSTANDING AND INTERPRETATION OF FACTOR ANALYSIS WHAT ARE THEIR LIMITATIONS?

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إلى أي مدى يمكن للطرق التي تستعمل فيـــها الجداول و الرسوم البيـــانية والأشكال الهندسية أن تساعد في فهم و بلورة دور العامل التحليلي في البحث العلمي.

Cet article essaie de montrer les méthodes qui utilisent les diagrammes, les figures géométriques et les représentations graphiques, pour comprendre et interpréter le rôle de facteur analytique dans une recherche scientifique.

Quelles sont donc ses limites ?

WHY FACTOR ANALYSIS IS USED BY RESEARCHERS?

The term factor analysis stands for a broad category of approaches to conceptualizing groupings (or clustering) of variables and even broader, collection of mathematical procedures for determining which variables belong to which groups.

Factor analysis, is used intensively by behavioral scientists because it allows them to determine the number and nature of the underlying variables among larger number of measures. It also helps them to extract common factor variances from sets of measures.

In fact this method tends to serve the cause of scientific parsimony; by reducing the multiplicity of tests and measures to greater simplicity.

Equally, it tells the researcher what tests or measures belong together, and which ones virtually measure the same thing and how much they do so. It is capable of reducing the number of variables to help the scientist concentrate on the main ones.

Overall, factor analysis is concerned with two main approaches:

The first approach deals particularly with the newly extracted factors and their algebraic representation, it includes:

- Component analysis, cluster analysis and factor analysis,
- The centroïd or unweighted summation extraction of factor components,
- The principal components or weighted summation extraction,
- Checking back from factor matrices to correlation matrices,
- And finally, the specification and estimation equation linking factors and variables.

- The second approach, deals with rotating factors and their geometric representation; it includes:

- The correlation coefficient geometrically represented,
- Plotting variables in hyperspace from the Vo matrix,
- The relation of cluster analysis to factor analysis and the effect of factor patterns.

THE PURPOSE OF THIS STUDY:

But the major concern of this study is to throw light on ways in which the use of diagrams, figures and graphs can aid the understanding and interpretation of factor analysis.

For this reason, the first geometrical representation to be considered by this study is the scattergram:

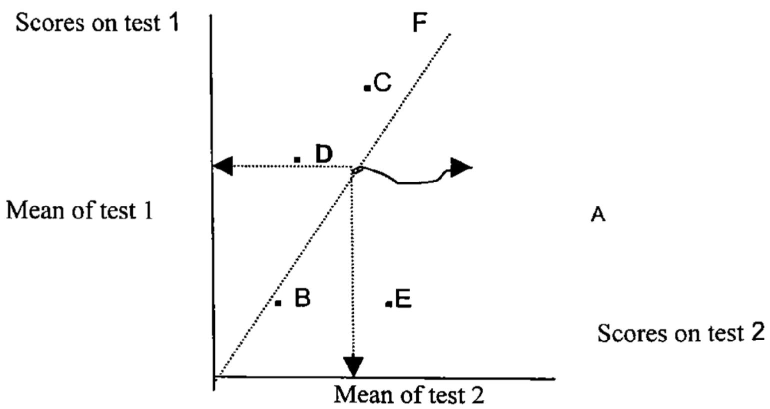


Figure 1: A scattergram

In fact, this "scatter-diagram" enables the researcher to depict a correlation by two coordinates, one for each variable, and allows him to plot the position of people thereon by their scores.

Thus, the first task of this scattergram is to show correlation in a way, which can aid the researcher to produce a geometrical

analysis and hence determine the way in which these variables are treated with greatest affinity.

In fact, this scattergram could be considered as very useful way to portray the correlation that exists between two variables. Furthermore, it shows how points are scattered, and how they represent individuals scores on test 1 (the vertical coordinate) and test 2 (the horizontal coordinate).

The elliptical shape shown by this scattergram, demonstrates that the point A is considered as the center of the ellipse as well as the a subject was tested and has got scores appropriate for point A, he also must have the sample mean score for test 1 and test 2. Thus, the points which are scattered on the graph give the researcher an idea of the way in which test scores are correlated.

The axis O F, which tends to divide the ellipse into two parts, enables the researcher to get an idea concerning the extent of scores on both tests; As for example, points B and C indicate that individuals represented by these points have obtained either low scores or high scores in both tests.

Scores of tests, which tend to cluster around O F have high positive correlation; whereas, the overall correlation appears to be very weak for individuals represented by points such as D whose scores are high on test 1 and low on test 2.

If however, points tend to cluster around a new axis created by D and E, the overall correlation will be negative. Moreover, to get an idea of the magnitude of the correlation, the researcher should join points of similar density.

In the following figure, one can see the single elliptical and circular contours, which belong to e series of concentric ellipses and circles showing the variation in density of points from the center; It is also possible to obtain a perfect correlation if all points are lined up on a straight line, this can be shown as follows:

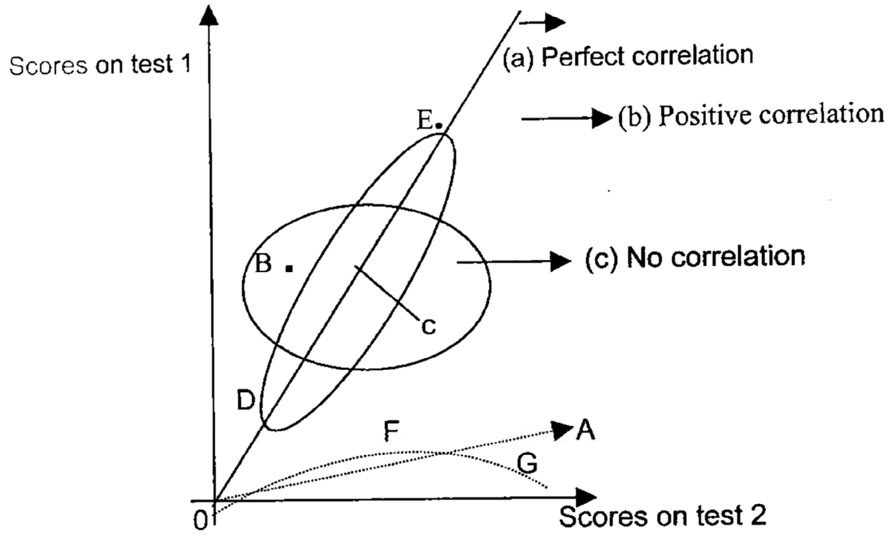


Figure 2: Contours for different correlations

As can be seen, this figure shows different points which are dispersed from the straight line, and which form an elliptical distributions.

In this situation, the correlation could become any value between +1 and zero. The shape of the ellipse indicates the size of the correlation; But, as the minor axis (B C) gets away fron the major axis, the correlation decreases and could attain zero. It is worth noticing here, that raw test scores from which correlations are derived should be distributed; but, in most cases, the researcher is allowed to use the original scores.

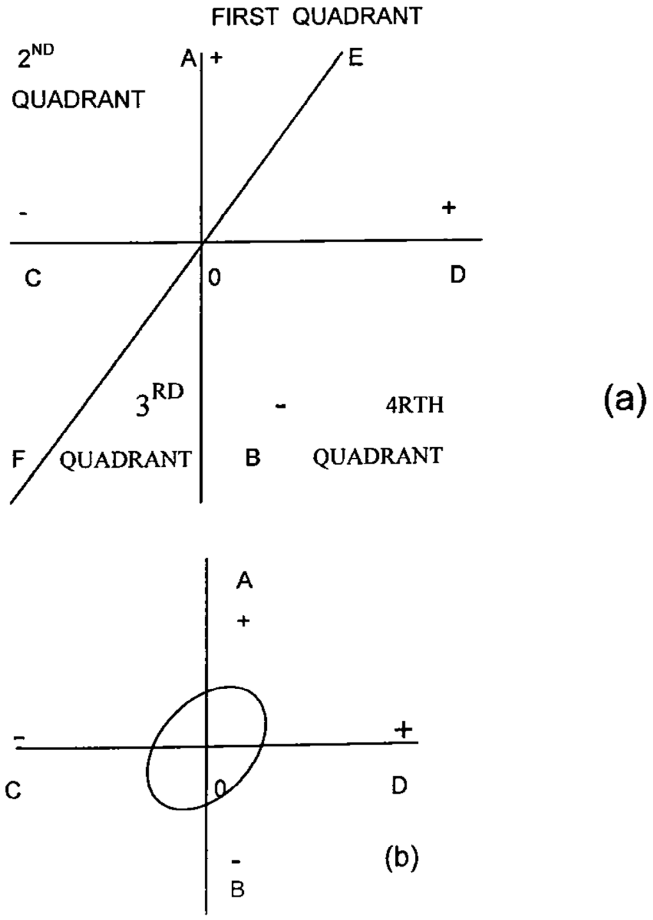
This geometrical picture has however one limitation impos. ' by the relationship that exists between the test scores. For example, in figure ', this relationship is shown mainly by a straight line (rectilinear relationship). Most points cluster around the axis of the ellipse.

But instead of being straight the line O F might bend toward one of the axes. If this relationship becomes curvilinear as it is the case in figure 2 (OFG), this might not be accepted by factor analysis, because in factor analysis factor determination is based on the assumption that correlations should be extracted from scores that can make linear relationships.

For more convenience, the researcher should covert the test scores; As for example, if he has obtained respectively 5 as the mean for a sample, 2 as the standard deviation and 7 the person's score, in this situation the standard score will be:

$$\frac{7-5}{2}=+1$$

The new standard scores will change the length of the major and minor axes of an elliptical distribution, and move the origin of the graph to the center of the ellipse. This can be shown on figure 3:



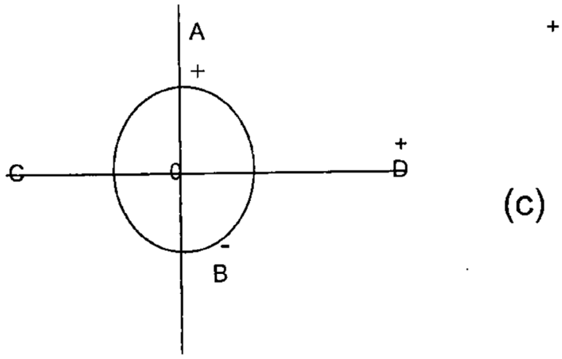


Figure 3: Contours on a scattergram using standard scores

OD and OA usually represent positive scores axes OC and OB represent negative scores of individuals who are higher than the mean on both tests are located in the first quadrant: Those whose scores are lower than the mean are represented in the third quadrant.

To increase the chance of producing a positive correlation, more individuals' scores are needed in quadrant 1 and 3. The line EF in figure 3 (a) indicates a perfect correlation because no part of the sample appears in the disparate quadrants 2 and 4. Figure 3 (b) shows that the ellipse contains larger numbers in quadrants 1 and 3 and tends to produce a positive relationship, whereas Figure 3 (c) shows that there is an equal number in all quadrants. Thus, it tends to produce a cancellation and to yield a zero correlation.

Moreover, an angle, which is situated, between two straight lines can express the correlation between two variables; This can be shown on the following figure:

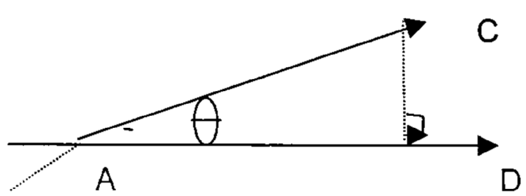


Figure 4: Test vectors

The lines AC and AB are vectors; they represent variables in both magnitude and direction. The relation between the two lines

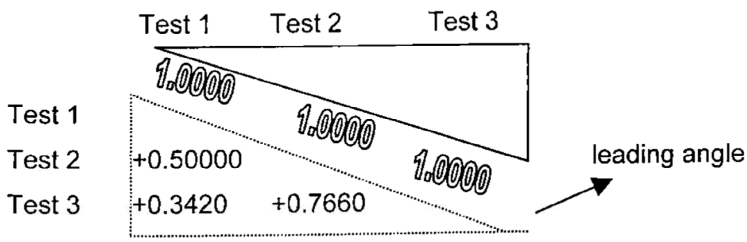
AC and AD and the cosine of the angle θ can be expressed.

In fact, by looking at the cosine table, one can obtain the value of the cosine of 60°, which is equal to 0.5000. Thus, the correlation between AC and AB (these two lines are equal in length) and the angle of 60° has reached 0.5000 in vector form.

Test vectors, which form an angle of 90° known as orthogonal produce zero correlation, because as the angle between the two vectors increases, the correlation between them decreases until it reaches zero. (For further evidence see table of cosines)

There is also another useful way for representing the intercorrelations of test scores; It is called a **correlation matrix**. This method allows the researcher to introduce a third set of test scores for the same sample. In this case, it is possible to correlate test 1 with test 2, test 1 with test 3 and test 2 with test 3.

If the values of these intercorrelations were 0.5000, + 3420 and + 0.7660, then they can be represented on the following correlation matrix:



A correlation matrix

It should be noted here that the repetition of the values of the intercorrelations in the upper triangle is worthless, because they are going to be the same.

Sometimes, two samples may take the same combination of tests; the upper triangle will be used for one sample and the lower triangle for the other. The loading diagonal shown in the matrix indicates that when a test is correlated with itself it gives a perfect value of 1.0.

Another important topic was raised by factor analysis. It concerns the existence of large number of test vectors. In situation like this, the researcher might get confused if he does not resort to a useful and economic way of expressing the complex associations of these test vectors. This method is called the centroïd method. This method was first used by Burt (1917) and was applied to the problem of determining a single general factor of the spearman type.

The complete centroïd method was developed by Thurstone (1931) in conjunction with the analysis of large batteries of psychological tests into several common factors, the name of the method connotes its close relationship to the mechanical concept of centroïd or center of gravity; but the centroïd form of analysis can best be described in geometric terms.

To illustrate, let us consider the example of the half open umbrella. The radiating frame gives the direction of test vectors and the handle is considered as a reference vector. The rest of the lines of reference needed for the resolution of test vectors can be thought of as common factor vectors. This can be illustrated in the following figure:

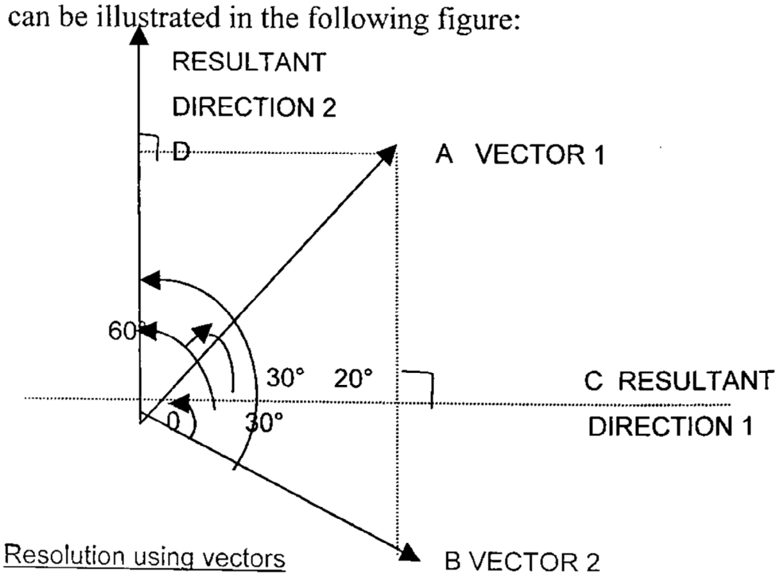


Figure 5: Resolution using vectors

This figure aids the researcher to understand both the centroïd method and the process of finding factors by resolving vectors; it shows two vectors of equal length OA and OB, which produce our angle of 60°.

By looking at a table of cosines this angle produces a correlation of + 0.5000, in factor analysis. The resultant vector 1 can be thought of as a reference factor from which the other vectors can be interpreted (centroïd method is being considered here).

It is also considered as a factor vector because it occupies a position between the vectors. The two vectors OA and OB are of equal length and the resultant 1 is bisecting them into two angles of 30° each.

Thus, it becomes possible to find the cosines of these two angles, which are situated between OA and OC and between OC and OB.

The resultant is used here as a reference factor; all vectors should be interpreted in terms of this resultant. It is worth noticing here that in factor analysis, the cosine of the angle produced by a test vector and factor vector (the resultant) is called a loading, which can be thought of as the correlation between the test variable and the factor vector.

This can be noticed clearly in figure 5 where the loading of the angles located between OA and OC is cos 30°=+0.8660. Similarly, figure 5 shows another reference line OD.

Thus, the angle located between OD and OA is 60°. It therefore, gives a loading of cos 60°=+0.8660. But the angle produced by vectors OD and OB has the value of 120° and gives a loading of cos 120°= - cos 60 = - 0.5000.

When these loadings are squared and added, they simply give a total of one . In other words, to calculate the loading of the first vector, the triangle OCA (see figure 5) was used together with the cosine of the angle COA produced by OC and OA.

The second loading is derived from the triangle ODA and the cosine of the angle DOA produced by OD and OA.

Thus, by squaring these two cosines the following result will be obtained : $\frac{OC^2}{OA^2} + \frac{OD^2}{OA^2} = \frac{OC^2 + OD^2}{OA^2}$

By using the (theorem of Pythagore) which says that in a triangle, the sum of two sides AC + OC is equal to the hypothenuse, the following result emerges:

$$OA^2 = OC^2 + AC^2 = OC^2 + OD^2$$

Then it is possible to substitute OA^2 in the equation (1) for $OC^2 + OD^2$ to finally obtain $\frac{OA^2}{OA^2} = \boxed{1}$

There are however some cases where it is possible to calculate first the correlations between tests, in order to find out the angles between the test vectors. This can be shown in the following matrix:

	T1	T2	T3	T4	T5
T1	-	10	70	90	100
T2	9848	-	60	80	90
T3	3420	5000	-	20	30
T4	0000	1736	9397	-	10
T5	-1736	0000	8660	9848	-

Upper triangle gives angles between vectors, lower gives corresponding correlations.

These values were used to find out the relation between vectors of five tests and the correct angles between them, this can be shown in the following figure:

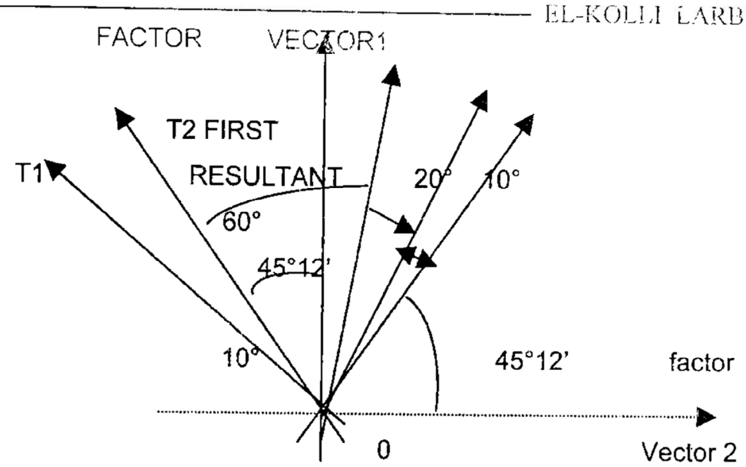


Figure 6: Vector diagram

In this figure, the cosine of an angle between vectors corresponds to the true value of the obtained correlations (see matrix above)

The researcher is bound to trace the first resultant (centroïd) from which the remaining vectors can be interpreted. Then, he must find out the relationship, which exists between all vectors and factor vector 1. This relationship is expressed by the values of the angles themselves; As for example, the angle, which measures 55°12', relates the vector T1 to the first factor vector.

Similarly, the angles that measure 45°12', 14°48', 34°48' and 44°48' relate respectively T2, T3, T4, T5 to the first factor vector.

The cosines of these angles give the correlation between test vectors and the factor vector. The researcher will discover by reference to cosine table that the first angle for T1 of 55°12' gives a cosine of +0.5707. Other values in the first column are discovered in the same manner; the same procedure will be used to find out the relationship between test vectors and the second factor vector.

Note that the squares of the loadings for any particular test will add up to one, this can be shown in the following table:

SHOW WAYS IN WHICH THE USE OF

LOADINGS OF TEST VECTORS IN FIGURE 6

	Factor loadings		Sum of squares of loadings for each test
	I	II	
1- Test 1	5707	-8211	1.0000
2- Test 2	7046	-7096	1.0000
3- Test 3	9668	2554	1.0000
4- Test 4	8211	5707	1.0000
5- Test 5	7096	7046	1.0000
Sum of squares of loadings for each factor	2.9347	2.0653	= 5.0000

There is also another important topic, which has to be raised here, that is of extracting factors. One should point out here that in factor analysis, only the common factors are required, and the methods employed rest upon assumptions as to how this has to be achieved?

Most factorists agree that Kaiser's criterion is a useful method for extracting factors. This criterion considers only factors having latent roots greater than 1, as common factors, the rest is taken out. This idea, however, has one limitation: For example, if the researcher is dealing with component analysis, this presents a disadvantage because it includes hybrid factors to be extracted. But these factors cannot be extracted, because the unique variance overlaps with common variance.

In this context, Cattell (1978) argued that in some cases some unique variance creeps into all factors, making the common variance too difficult to be altered.

Thus, it becomes very important to identify the optimum number of factors that can be taken out, before it will be very difficult to alter the common variance. Cattell (1978) proposed the use of the scree test. It was given the name scree (the test) because it is like the straight scree of rock debris, which a mountaineer often sees running out of a steady angle of debris stability from the foot of a mountain. This can be shown in the following figure:

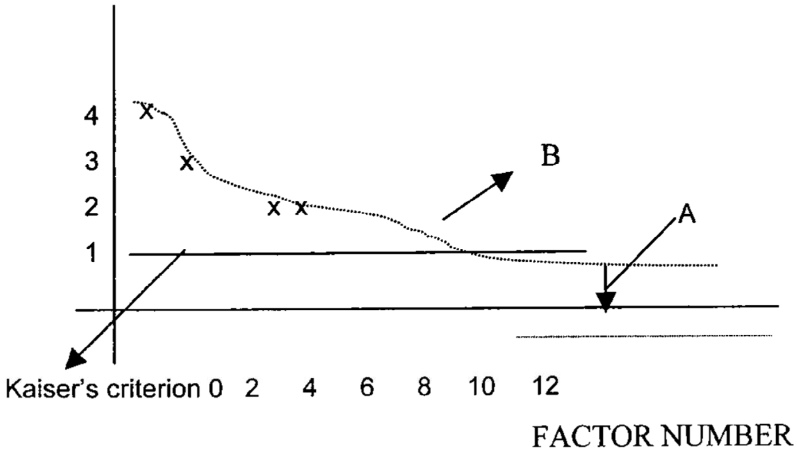


Figure 7: The scree test for more than 15 variables

As can be seen, latent roots (the axis OY) show that the number of factors that are greater than unity are considered as common factors (9). The remaining factors are below unity and need to be taken out; but by using Cattell's (1978) scree test 4 more factors will be qualified than Kaiser's criterion. By using Kaiser's criterion, only the first nine factors would have been accepted.

The following figure also is very important in helping the researcher understand factor analysis. It concerns particularly the effects of factor rotation on factor patterns.

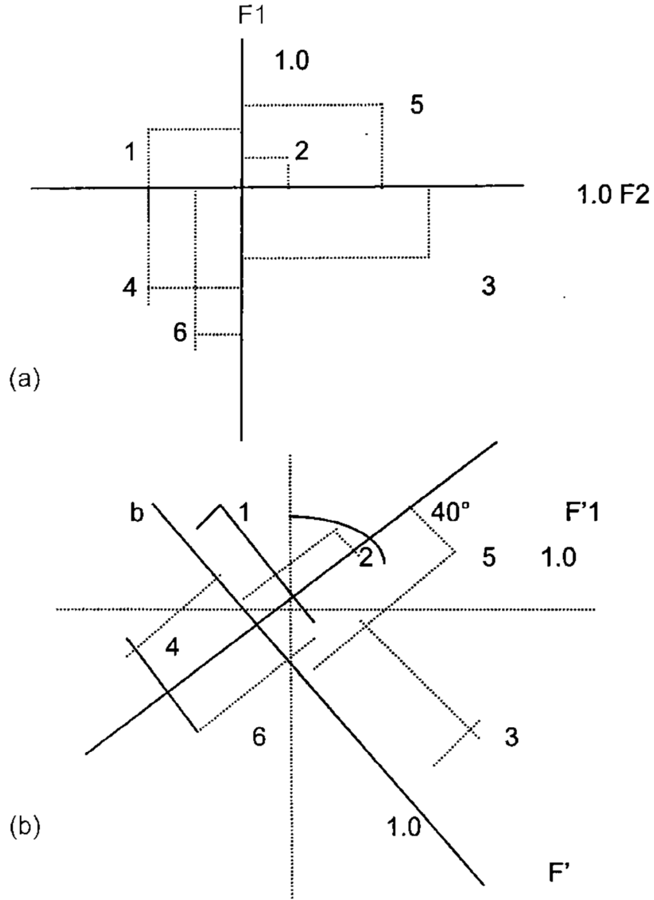


Figure 8: Effect of rotaring factor axes in a 40° clockwise shift, illustrating invariant configuration with changing loadings

This figure shows that in factor analysis the researcher is free to spin the axes to get some new positions.

The projection of variables on them (the axes) will change as the researcher spins, despite the fact that all positions are equivalent in accounting for the correlation.

Despite the rotation of the factor vector, the figure shows that the angular relations among variables remain the same. But the projection of the variables on these new axes F'1 and F'2 are altered by the rotation and if one measures them carefully he will find the new projections

To calculate the new projections F'1 and F'2 from the prerotated projections F'1 and F'2, a well known trigonometrical formula can be rised:

F'1 = F'2 Cos Θ + F2 Sin Θ

F'2 = F'2 Cos Θ - F2 Sin Θ

Where Θ is the angle of shift.

The following figure also might help the researcher understand the role of factor rotation in factor analysis.

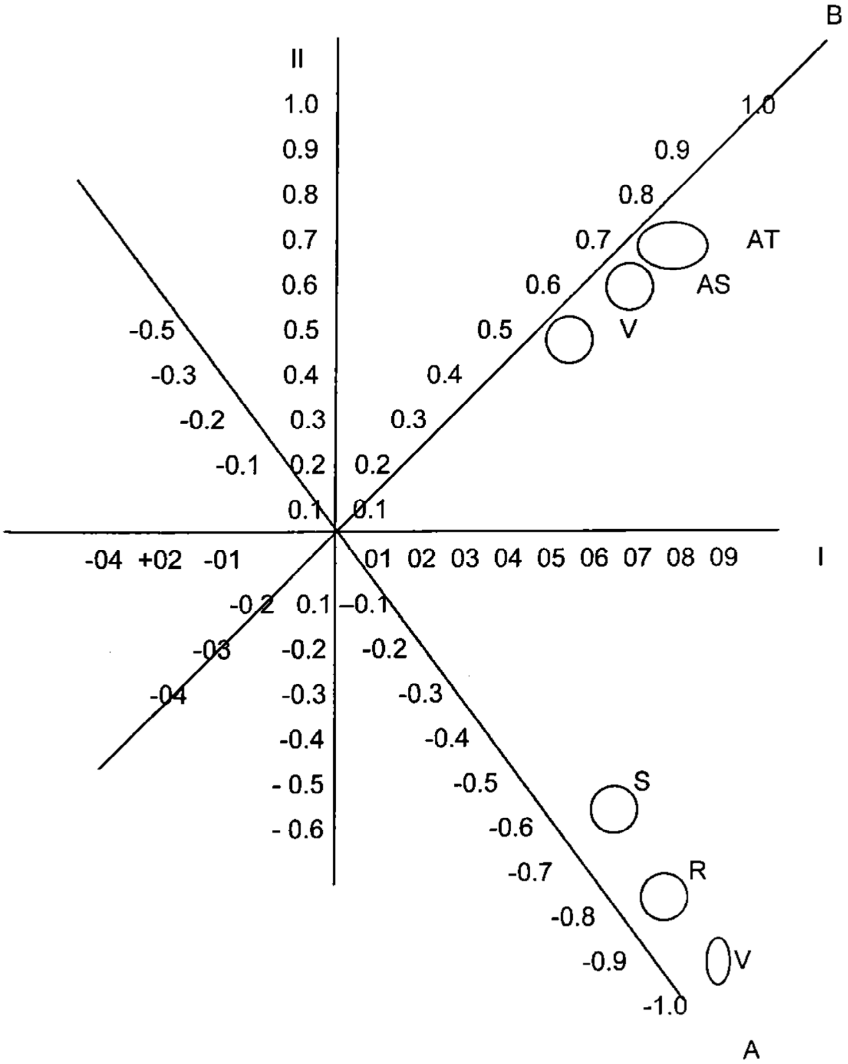


Figure 9: The rotation of two factors

If the researcher plots the loadings of I and II, he will notice the existence of the original unrotated structure.

Then, he is allowed to surving the axes so that I goes as near as possible to the V, R and S and at the same time II goes as near as possible to the V, AS and AT points. By so doing, the researcher is aiming to form a rotation of 45 degree.

In fact this graph aids the researcher understand that factor analyst searches for the unities that presumably underlines test performances.

Similarly, this rotation enables the researcher to search out the relations among variables in multidimensional factor space. The researcher also knows that this graph will help him to get through knowledge of the empirical relations among tests or other measures.

Finally, it enables him to probe in factor space with reference axes until he finds the unities or relations among relations. In some cases, however, the researcher is bound to rotate the axes through different angles to finally arrive at an oblique rotation.

But this method is becoming a laborious technique. Thus, a hand rotation is no longer an efficient method to get through different types of rotations. Computer facilities are now widely used and are enabling most researchers to rely on **analytic rotation,** because it aids them to obtain computerized solutions using mathematical approximations.

The following figures raises another important topic: the orthogonal rotation of factors:

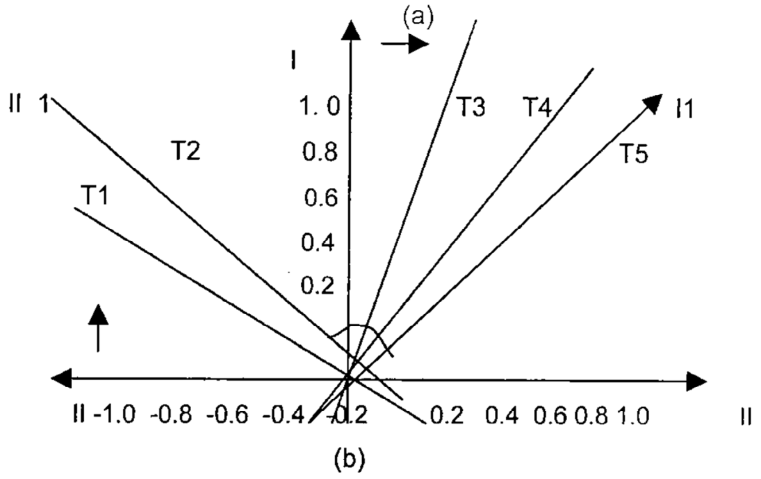


Figure 10: The orthogonal rotation of the factors

The vectors and the angles of this figure are closely linked to the values displayed in the following diagram, which shows the arrangement when the loadings of factor I are plotted against factor II. The factor vectors are acting as reference vectors:

	Factor loadings	
	I	II
Test 1	57	-82
Test 2	71	-71
Test 3	97	26
Test 4	82	57
Test 5	71	71

To make the rotation of vectors possible (see the figure 10) the researcher should rotate the reference axes I and II clockwise and should try to keep them at 90° to each other. By so doing the rotation of I to II, has brought T1 to T2 close to the hyper plane of I, which becomes now a new position of II, in an orthogonal rotation. (Hyper plane means three or more dimensional arrays, which fall at right angles to the reference vector).

In this example, the rotation was stopped immediately after T2 and T5 have formed an angle of 90° and after factor I passed through the point T2 and factor II hyper plane passed through T5.

This position was considered as a first approximation and as the most mathematically convenient.

One should notice here that with three or more factors to deal with, the performing of a graphical or analytical solution should go throw stages depending on the number of factors involved.

If the researcher has an experienced hand and eyes to make these graphical hand rotations an effective technique, the task of rotating the axes can become a marathon project. But on the whole, they (the axes and vectors) tend to aid the researcher to understand the relations among variables in a multidimensional space.

CONCLUSION:

In the end one should point out that these figures, diagrams and graphs had made an expressive contribution in aiding the researcher to understand factor analysis.

Despite some of their limitations or imperfections they have, however, proved to be a serviceable and productive tools at disposal of the researcher.

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